A Very Basic Introduction to R – Part III

Simulation

The `runif()` function can be used to simulate n independent uniform random variables.

For example, we can generate 5 uniform random numbers on $[0, 1]$ as follows:

```
> runif(5)
```

```
[1] 0.74816856 0.67462803 0.03878417 0.90323527 0.33053944
```

In order to generate uniform numbers on an interval of the form $[a, b]$, we use the arguments `min=a` and `max=b`. For example,

```
> runif(3, 1.2, 5.8)
```

```
[1] 5.777474 4.995633 2.492278
```

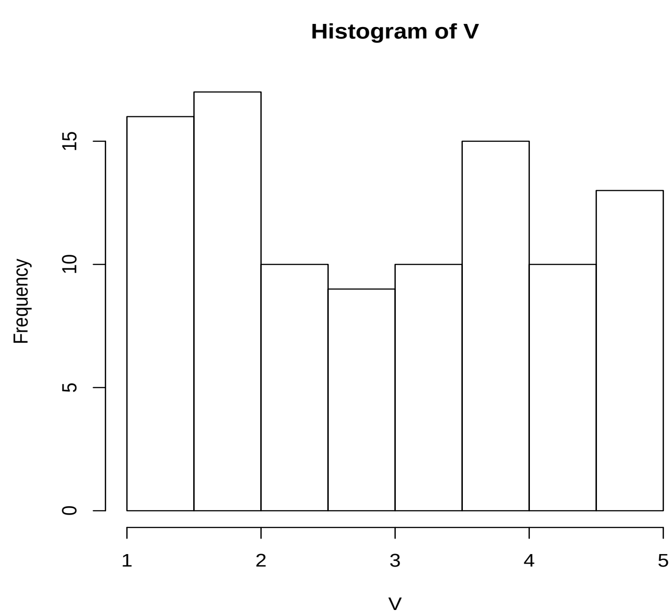
gives 3 uniform numbers on $[1.2, 5.8]$.

In the next example, we will assign 100 independent uniform numbers on the interval $[1, 5]$ to a vector object called `V`.

```
> V <- runif(100, 1, 5)
```

We can plot a histogram of the numbers:

```
> hist(V)
```



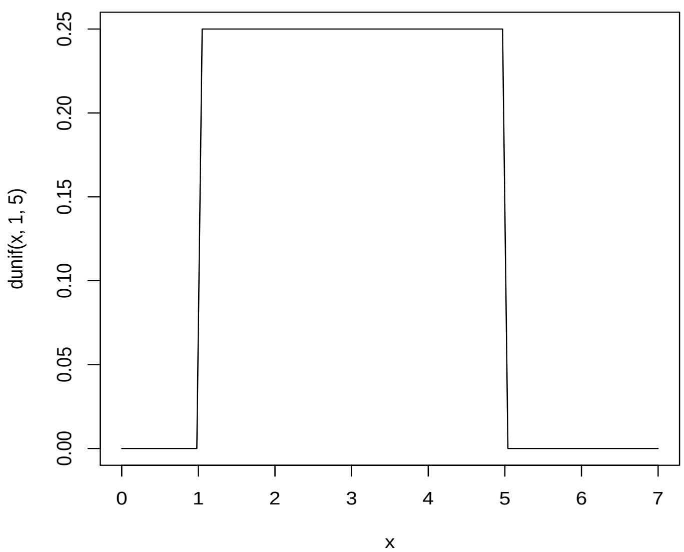
The histogram is an estimate of the probability density function, $f(v)$. The probability density function of the uniform random variable is given by

$$f(v) = 1/(b - a)$$

for $v \in [a, b]$, and 0, otherwise. It can be calculated using the `dunif()` function.

For example,

```
> curve(dunif(x, 1, 5), from=0, to=7)  # uniform pdf on [1,5]
```



We can calculate the mean, standard deviation and variance of the sample in `V`:

```
> mean(V)

[1] 2.90512

> sd(V)

[1] 1.2235

> var(V)

[1] 1.496953
```

The sample mean is an estimate of the population mean (or expected value). The expected value of a uniform random variable is given by

$$E[V] = \int_a^b v f(v) dv = \int_a^b \frac{v}{b-a} dv = \frac{v^2}{2(b-a)}|_a^b = \frac{b^2 - a^2}{2(b-a)} = \frac{a+b}{2}$$

Therefore, the expected value of the uniform random variable on $[1, 5]$ is $(1+5)/2 = 3$. Compare this value with the sample mean calculated from the simulated sample.

The variance of a random variable is calculated from the formula:

$$\mathrm{Var}(X) = E[X^2] - E[X]^2$$

where

$$E[X^2] = \int_a^b x^2 f(x) dx = \int_a^b \frac{x^2}{b-a} dx = \frac{x^3}{3(b-a)}|_a^b = \frac{b^3 - a^3}{3(b-a)}.$$

Using algebra, we can show that

$$\mathrm{Var}(X) = \frac{(b-a)^2}{12}.$$

For the uniform random variable on $[1, 5]$, the variance is $16/12$ or 1.333. Compare this with the sample variance calculated for the simulated sample.

The standard deviation is defined as the square root of the variance. Therefore, the standard deviation of the uniform random variable on $[1, 5]$ is $\sqrt{1.333}$ or

```
[1] 1.154701
```

Compare this value with the sample standard deviation calculated for the simulated sample above.

Exercises

1. Generate 20 uniform random numbers on the interval $[0.5, 2.5]$, and plot a histogram of the resulting sample.
2. Calculate the expected value and variance of a uniform random number on $[0.5, 2.5]$.
3. Assign 2000 uniform random numbers on the interval $[-1.7, -1.3]$ to a vector called `U`.
4. Calculate the mean value of `U`. Compare with the theoretical value.
5. Calculate the variance of `U`. Compare with the theoretical value.
6. Plot the histogram of the sample in `U`. Compare with a plot of the theoretical probability density function.
7. Compute the theoretical value of $E[U^3]$, and compare with the value of `mean(U^3)`.
8. Compute the theoretical value of $E[U^4]$, and compare with the value of `mean(U^4)`.
9. Repeat questions 3 through 8 for `W`, a vector of 10000 uniform random numbers on the interval $[3, 7]$.
10. Compute the theoretical value of $E[\sqrt{W}]$ and compare with the value of `mean(sqrt(W))`.