## A Very Basic Introduction to R – Part V

### Simulation of Poisson Random Variables

The `rpois()` function can be used to simulate $N$ independent Poisson random variables.

For example, we can generate 15 Poisson random numbers with parameter $\lambda = 4$ as follows:

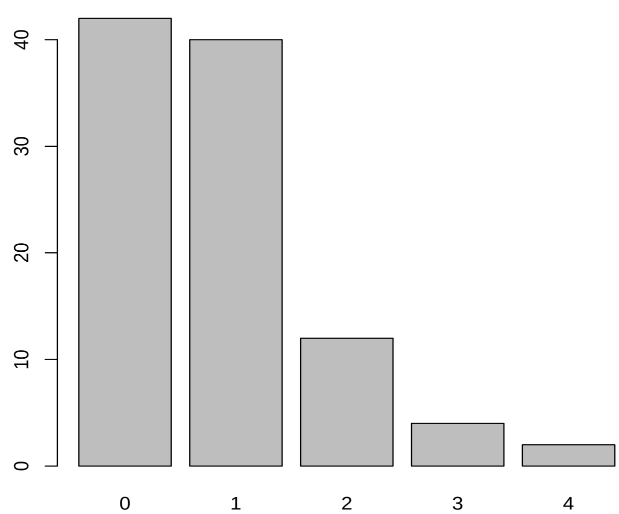
```
> rpois(15, 4)

 [1] 4 3 3 3 4 4 6 7 2 3 3 7 9 4 4
```

In the next example, we will assign 100 independent Poisson numbers with parameter $\lambda = 0.7$ to a vector object called `V`.

```
> V <- rpois(100, 0.7)
```

We can plot a bar chart of the numbers:

```
> barplot(table(V))
```




The bar plot is an estimate of the probability distribution $P(V = v)$. It is more appropriate than a histogram, because the data are discrete, not continuous.

The theoretical distribution is

$$P(V = v) = \frac{\lambda^v e^{-\lambda}}{v!}$$

for $v = 0, 1, 2, \ldots$, and 0, otherwise. It can be calculated using the `dpois()` function.

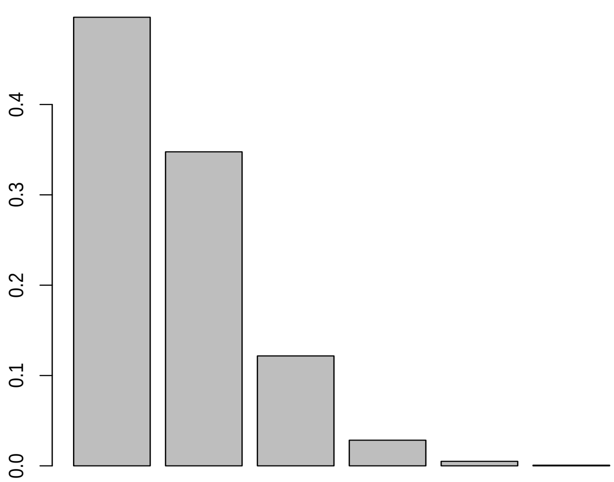
For example, we can calculate $P(V = 3)$ where $V$ is a Poisson random variable with parameter $\lambda = 2$, using

```
> dpois(3, 2)
```

```
[1] 0.180447
```

A bar chart of the distribution of the Poisson random variable with parameter $\lambda = .7$ can be obtained using

```
> barplot(dpois(0:5, 0.7))
```




We can calculate the mean, standard deviation and variance of the sample in `V`:

```
> mean(V)
```

```
[1] 0.84
```

```
> sd(V)
```

```
[1] 0.9289942
```

```
> var(V)

[1] 0.8630303
```

The sample mean is an estimate of the population mean (or expected value). The expected value of a Poisson random variable is given by

$$E[V] = \lambda$$

The variance of a random variable is calculated from the formula:

$$\mathrm{Var}(X) = \lambda.$$

In other words, the mean and variance are equal for Poisson random variables.

The standard deviation is $\sqrt{\lambda}$.

For the Poisson random variable with $\lambda = 2.25$, we can calculate these quantities by hand or by using R:

```
> lambda <- 2.25
> lambda # Expected Value, and Variance

[1] 2.25

> sqrt(lambda) # Standard Deviation

[1] 1.5
```

We can compare these with what we would obtain from a simulated sample of 10000 binomial random variables:

```
> P <- rpois(10000, 2.25)
> mean(P) # sample mean

[1] 2.246

> var(P) # sample variance

[1] 2.220906

> sd(P) # sample standard deviation

[1] 1.49027
```

## Exercises

1. Generate 20 Poisson random numbers with $\lambda = 1.9$, and plot a bar chart of the resulting sample.
2. Calculate the expected value and variance of the above numbers.
3. Assign 2000 Poisson random numbers with parameter $\lambda = 4.5$ to a vector called `U`.
4. Calculate the mean value of `U`. Compare with the theoretical value.
5. Calculate the variance of `U`. Compare with the theoretical value.
6. Plot the bar chart of the sample in `U`. Compare with a plot of the theoretical probability distribution.
7. Use simulation to estimate the expected value of $U^3$ where $V$ is Poisson with parameter $\lambda = 1.5$.
8. Suppose $V$ is a Poisson random variable with mean 10. Simulate 10000 values of $V$ and compute their mean and variance, comparing with the theoretical values.
9. Simulate 200 Poisson numbers with parameter $\lambda = 17$. Find their sample mean and variance. Compare with the theoretical values.