First Steps to R

W. John Braun, UBC

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An Overview of R





These lectures introduce R, as originally developed as S, by John *Chambers* and others at Bell Laboratories in 1976, and implemented and made into an Open Source program by Robert *Gentleman* and Ross *Ihaka* in 1995.

As you learn R, there is nothing wrong with making errors when learning a programming language like R.

You learn from your mistakes, and there is no harm done.

Try out the code embedded into the accompanying text and experiment with new variations to discover how the system will respond.

¹ https://www.r-project.org/Licenses/GPL-2



R can be downloaded for free from CRAN*.

A *binary version* is usually simplest to use and can be installed in Windows and Mac fairly easily.

A binary version is available for Windows from the web page http://cloud.r-project.org/bin/windows/base.

The "setup program" setup is usually a file with a name like R-3.6.1-win.exe.

Clicking on this file will start an almost automatic installation of the R system. Clicking "Next" several times is often all that is necessary in order to complete the installation.

^{*}http://cloud.r-project.org



An R icon (\mathbf{R}) will appear on your computer's desktop upon completion.

RStudio is also very popular. You can download the "Open Source Edition" of "RStudio Desktop" from http://www.rstudio.com/, and follow the instructions to install it on your computer.

Although much or all of what is described here can be carried out in RStudio, there will be little further comment about that environment.

Thus, you might find that some of the instructions to be carried out at the command line can also be carried out with the menu system in RStudio.



Clicking on the R icon, or opening RStudio similarly, should provide you with access to a window or pane, called the *R console* in which you can execute commands.



The > sign is the R prompt which indicates where you can type in the command to be executed.



You can do arithmetic of any type, including multiplication:

| braun@PeterHall: ~/.wine/drive_c/Program Files/R/R-3.6.1/bin/i386 |
|--|
| R version 3.6.1 (2019-07-05) "Action of the Toes" Copyright (C) 2019 The R Foundation for Statistical Computing Platform: i386-w64-mingw32/i386 (32-bit) |
| R is free software and comes with ABSOLUTELY NO WARRANTY. You are welcome to redistribute it under certain conditions. Type 'license()' or 'licence()' for distribution details. |
| Natural language support but running in an English locale |
| R is a collaborative project with many contributors. Type 'contributors()' for more information and 'citation()' on how to cite R or R packages in publications. |
| Type 'demo()' for some demos, 'help()' for on-line help, or 'help.start()' for an HTML browser interface to help. Type 'q()' to quit R. |
| > 1111 + 1234 |

By hitting the "Enter" key, you are asking R to execute this calculation.



The answer appears on the next line:



Often, you will type in commands such as this into a script window, as in RStudio, for later execution, through hitting "ctrl-R" or another related keystroke sequence.

Executing commands in R



Objects that are built in to R or saved in your *workspace*, i.e. the environment in which you are currently doing your calculations, can be displayed, simply by invoking their name.

women

>

For example,

the data set or *data frame* called women contains information on heights and weights of American women:

height weight ## ## ## 3 ## ## 5 ## ## ## 8 ## 9 ## ## ## ## ##



Packages

One of the major strengths of R is the availability of add-on *packages* that have been created by statisticians and computer scientists from around the world.

There are thousands of packages, e.g. graphics, ggplot2, and MPV.

A package contains functions and data which extend the abilities of R.

Every installation of R contains a number of packages by default (e.g. *base*, stats, and *graphics*) which are automatically loaded when you start R.



Packages

To load an additional package, for example, called DAAG, type

library(DAAG)

If you get a warning that the package is can't be found, then the package doesn't exist on your computer, but it can likely be installed. Try

```
install.packages("DAAG")
```



In RStudio, it may be simpler to use the Tools menu.





Choose "Install Packages":

| S. | RStudio | |
|--|--|---------------------|
| <u>File E</u> dit <u>C</u> ode <u>V</u> iew <u>P</u> lots <u>S</u> ession <u>B</u> uild <u>D</u> ebug <u>P</u> rofile <u>T</u> | ools <u>H</u> elp | |
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| R version 3.4.4 (2018-03-15) "Someone to Lean On" | Data | |
| phoon Platform: x86 64-pc-linux-gnu (64-bit) | <pre>O lob.lme List of 18</pre> | ٩ = |
| | | |
| R is free software and comes with ABSOLUTELY NO WARRANTY. | | |
| 2 Type 'license()' or 'licence()' for distribution details. | | |
| 3 | | |
| 4 Natural language support but running in an English locale | | |
| 6 R is a collaborative project with many contributors. | | |
| 7 Type 'contributors()' for more information and | | |
| <pre>8 'citation()' on how to cite R or R packages in publications.</pre> | | |
| 9 Type 'demo()' for some demos, 'help()' for on-line help. or | | |
| 10 'help.start()' for an HTML browser interface to help. Install from: | 2 Configuring Repositories | |
| Type 'q()' to quit R. Repository (CRA | N) viewer | |
| 13 [Workspace loaded from ~/ RData] | Export - 😳 🧹 | G |
| 14 Packages (separ | rate multiple with space or comma): | |
| 15 > | | |
| 16 17 Install to Library | | |
| 18 /home/braun/R/x/ | 86_64-pc-linux-gnu-library/3.4 [Default] | |
| 19 | | |
| 20 🗸 Install depen | dencies | |
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Type in the name of the package you are requesting, and click "Install":





Packages

Once DAAG is installed, it can be loaded using the <code>library()</code> function, and you can access data frames and functions that were not available previously. For example, the <code>seedrates</code> data frame is now available:

| seedrates |
|-----------|
|-----------|

| ## | | rate | grain |
|----|---|------|-------|
| ## | 1 | 50 | 21.2 |
| ## | 2 | 75 | 19.9 |
| ## | 3 | 100 | 19.2 |
| ## | 4 | 125 | 18.4 |
| ## | 5 | 150 | 17.9 |



Using one object from a package at a time

The *MPV* package is installed on my system, but I have not loaded it. I only want to access the p2.12 data frame and nothing else.

To do this, just type the package name (MPV), followed by two colons (::) and the object name you seek.

| MP۱ | /::p | 2.12 | |
|-----|------|------|--------|
| ## | | temp | usage |
| ## | 1 | 21 | 185.79 |
| ## | 2 | 24 | 214.47 |
| ## | 3 | 32 | 288.03 |
| ## | 4 | 47 | 424.84 |
| ## | 5 | 50 | 454.68 |
| ## | 6 | 59 | 539.03 |
| ## | 7 | 68 | 621.55 |
| ## | 8 | 74 | 675.06 |
| ## | 9 | 62 | 562.03 |
| ## | 10 | 50 | 452.93 |
| ## | 11 | 41 | 369.95 |
| ## | 12 | 30 | 273.98 |



Calculations in R

You can control the number of digits in the output with the options () function.

This is useful when reporting final results such as means and standard deviations, since including excessive numbers of digits can give a misleading impression of the accuracy in your results.

| | 583/31 | | options (digits=3) |
|---------|-----------------|------|--------------------|
| Compare | ## [1] 10 00C/F | with | 583/31 |
| | ## [I] I0.00043 | | ## [1] 18.8 |

Calculations in R



Observe the patterns in the following calculations.

```
options (digits = 18)
11111111*111111
## [1] 1234567654321
11111111*111111
## [1] 123456787654321
111111111*111111
## [1] 12345678987654320
```

The error in the final calculation is due to the way R stores information about numbers.

There are around 17 digits of numeric storage are avaiable.



Most data sets are stored in R as *data frames*, such as the women object we encountered earlier.

Data frames are like matrices, but where the columns have their own names.

You can obtain information about a built-in data frame by using the help() function. For example, observe the outcome to typing help(women).

It is generally unwise to inspect data frames by printing their entire contents to your computer screen, as it is far better to use graphical procedures to display large amounts of data or to exploit numerical summaries.



The *summary()* function provides information about the main features of a data frame:

summary (women)

| ## | height | weight |
|----|--------------|-------------|
| ## | Min. :58.0 | Min. :115 |
| ## | 1st Qu.:61.5 | 1st Qu.:124 |
| ## | Median :65.0 | Median :135 |
| ## | Mean :65.0 | Mean :137 |
| ## | 3rd Qu.:68.5 | 3rd Qu.:148 |
| ## | Max. :72.0 | Max. :164 |

Columns can be of different types from each other. An example is the built-in chickwts data frame:

summary(chickwts)

| ## | weigh | f | eed | |
|----|-----------|----|-----------|------|
| ## | Min. :1 | 08 | casein | :12 |
| ## | 1st Qu.:2 | 04 | horsebear | n:10 |
| ## | Median :2 | 58 | linseed | :12 |
| ## | Mean :2 | 61 | meatmeal | :11 |
| ## | 3rd Qu.:3 | 24 | soybean | :14 |
| ## | Max. :4 | 23 | sunflowe | r:12 |

One column is of factor type while the other is numeric.



Data frames

If you want to see the first few rows of a data frame, you can use the *head()* function:

head(chickwts)

| ## | | weight | feed |
|----|---|--------|-----------|
| ## | 1 | 179 | horsebean |
| ## | 2 | 160 | horsebean |
| ## | 3 | 136 | horsebean |
| ## | 4 | 227 | horsebean |
| ## | 5 | 217 | horsebean |
| ## | 6 | 168 | horsebean |

The *tail()* function displays the last few rows.



The number of rows can be determined using the *nrow()* function:

nrow(chickwts)

[1] 71

Similarly, the *ncol()* function counts the number of columns.



The str() function is another way to extract information about a data frame:

str(chickwts)



If you have prepared the data set yourself, you could simply type it into a text file, for example called mydata.txt, perhaps with a header indicating column names, and where you use blank spaces to separate the data entries.

The *read.table()* function will read in the data for you as follows:

mydata <- read.table("mydata.txt", header = TRUE)</pre>

The object mydata now contains the data read in from the external file.



You could use any name that you wish in place of mydata, as long as the first element of its name is an alphabetic character.

If the data entries are separated by commas and there is no header row, as in the file $wx_13_2006.txt$, you would type:

wx1 <- read.table("wx_13_2006.txt", header=F, sep=",")</pre>



Often, your data will be in a spreadsheet.

If possible, export it as a .csv file and use something like the following to read it in.

If you cannot export to .csv, you can leave it as .xlsx and use the *read.xslx()* command in the *xlsx* package (Dragulescu and Arendt, 2018).



When reading in a file with columns separated by blanks with blank missing values, you can use code such as

```
dataset1 <- read.table("file1.txt", header=TRUE,
    sep=" ", na.string=" ")
```

This tells R that the blank spaces should be read in as missing values.

Observe the contents of dataset1:

| dat | cas | set1 | _ | | |
|-----|-----|------|----|-----|--|
| ## | | Х | У | Z | |
| ## | 1 | 3 | 4 | NA | |
| ## | 2 | 51 | 48 | 23 | |
| ## | 3 | 23 | 33 | 111 | |

Note the appearance of *NA*. This represents a *missing value*. Functions such as is.na() are important for *detecting missing values* in vectors and data frames.

For more information about handling of missing values, check out the See Also section of help(is.na) and the *mice* package (van Buuren and Groothuis-Oudshoorn, 2011).



Sometimes, external software exports data files that are tab-separated. When reading in a file with columns separated by tabs with blank missing values, you could use code like

```
dataset2 <- read.table("file2.txt", header=TRUE,
    sep="\t", na.string=" ")
```

| | dataset2 | | | | | | |
|----------------------------|----------|---|----|-----|----|--|--|
| | ## | | Х | У | Z | | |
| Again, observe the result: | ## | 1 | 33 | 223 | NA | | |
| | ## | 2 | 32 | 88 | 2 | | |
| | ## | 3 | 3 | NA | NA | | |

If you need to skip the first 3 lines of a file to be read in, use the skip=3 argument.





To extract the height column from the women data frame, use the \$ operator:

| women\$ | heig | nt | | | | | | | | | | | | | | |
|---------|------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|--|
| ## [1 |] 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | |



If you want only the chicks who were fed horsebean, you can apply the *subset()* function to the chickwts data frame:

```
chickHorsebean <- subset(chickwts, feed == "horsebean")
chickHorsebean</pre>
```

| ## | | weight | feed |
|----|----|--------|-----------|
| ## | 1 | 179 | horsebean |
| ## | 2 | 160 | horsebean |
| ## | 3 | 136 | horsebean |
| ## | 4 | 227 | horsebean |
| ## | 5 | 217 | horsebean |
| ## | 6 | 168 | horsebean |
| ## | 7 | 108 | horsebean |
| ## | 8 | 124 | horsebean |
| ## | 9 | 143 | horsebean |
| ## | 10 | 140 | horsebean |



You can now calculate the mean and standard deviation, and so on, of these weights:

```
mean(chickHorsebean$weight) # mean
## [1] 160.2
sd(chickHorsebean$weight) # standard deviation
## [1] 38.626
```



In order to extract the 4th row from the chickHorsebean data frame, type

chickHorsebean[4,]
weight feed
4 227 horsebean

To extract the element in the 2nd column of the 7th row of women, type

```
women[7, 2]
## [1] 132
```

If we want the elements in the 4th through 7th row of the 2nd column of women, we can use

women[4:7, 2] ## [1] 123 126 129 132

Note the use of the : operator:

4:7

[1] 4 5 6 7



Another built-in data frame is airquality.

If we want to compute the mean for each of the first sapply(4 columns of this data ## Oz frame, we can use the ## sapply() function:

sapply(airquality[, 1:4], mean)
Ozone Solar.R Wind Temp
NA NA 9.9575 77.8824

The sapply() function applies the same function to all columns of the supplied data frame.



Factors offer an alternative, often more efficient, way of storing character data.

For example, a factor with 6 elements and having the two levels, control and treatment can be created using:factor()

```
grp <- c("control", "treatment", "control", "treatment",</pre>
    "treatment", "control")
qrp
## [1] "control" "treatment" "control" "treatment"
## [5] "treatment" "control"
qrp <- factor(qrp)</pre>
qrp
## [1] control treatment control treatment treatment
## [6] control
## Levels: control treatment
```



Consider the built-in data frame InsectSprays

summary(InsectSprays)

| ## | count | spray |
|----|--------------|-------|
| ## | Min. : 0.0 | A:12 |
| ## | 1st Qu.: 3.0 | B:12 |
| ## | Median : 7.0 | C:12 |
| ## | Mean : 9.5 | D:12 |
| ## | 3rd Qu.:14.2 | E:12 |
| ## | Max. :26.0 | F:12 |

The second column of this data frame is a *factor* representing the different types of spray used in the associated experiment.

Factors



The levels of a factor can be listed using the *levels()* function:

| levels (InsectSprays\$spray) | | | | | | | |
|-------------------------------------|-----|-----|-----|-----|-----|-----|-----|
| ## | [1] | "A" | "B" | "C" | "D" | "E" | "F" |



Factors are a more efficient way of storing character data when there are repeats among the vector elements.

This is because the levels of a factor are internally coded as integers.

To see what the codes are for the spray factor, we can type

The labels for the levels are only stored once each, rather than being repeated.



We can change the labels for the factor using the *levels()* function as follows:

```
levels(InsectSprays$spray)[3] <- "Raid"</pre>
```

Observe the effect of the change in

| <pre>summary(InsectSprays\$spray)</pre> | | | | | | | |
|---|----|----|------|----|----|----|--|
| ## | A | В | Raid | D | E | F | |
| ## | 12 | 12 | 12 | 12 | 12 | 12 | |



The ${\tt levels}$ () function also offers a simple way to collapse categories.

Suppose we are interested in comparing the first three levels with the last three levels.

We can create a new factor for this purpose as follows:

```
InsectSprays$newFactor <- InsectSprays$spray
levels(InsectSprays$newFactor) <- c("A", "A", "A",
    "B", "B", "B")</pre>
```



Check the result:

summary (InsectSprays)

| ## | count | spray | newFactor | |
|----|--------------|---------|-----------|--|
| ## | Min. : 0.0 | A :12 | A:36 | |
| ## | 1st Qu.: 3.0 | в :12 | B:36 | |
| ## | Median : 7.0 | Raid:12 | | |
| ## | Mean : 9.5 | D :12 | | |
| ## | 3rd Qu.:14.2 | E :12 | | |
| ## | Max. :26.0 | F :12 | | |



A *tibble* can be created from an existing data frame, using the <code>as_tibble()</code> function, found in the *tibble* package (Wickham, 2017).

library(tibble) # install.packages("tibble"), if needed
trees.tbl <- as_tibble(trees) # trees is a data frame</pre>

Tibbles



Tibbles are like data frames, but they prevent you from doing silly things, like printing a whole data set to the screen:

| <pre>trees.tbl # trees.tbl is a tibble</pre> | | | | | | | | |
|--|-----|-------------|-------------|-------------|--|--|--|--|
| | | | | | | | | |
| ## # A tibble: 31 x 3 | | | | | | | | |
| ## | | Girth | Height | Volume | | | | |
| ## | | <dbl></dbl> | <dbl></dbl> | <dbl></dbl> | | | | |
| ## | 1 | 8.3 | 70 | 10.3 | | | | |
| ## | 2 | 8.6 | 65 | 10.3 | | | | |
| ## | 3 | 8.8 | 63 | 10.2 | | | | |
| ## | 4 | 10.5 | 72 | 16.4 | | | | |
| ## | 5 | 10.7 | 81 | 18.8 | | | | |
| ## | 6 | 10.8 | 83 | 19.7 | | | | |
| ## | 7 | 11 | 66 | 15.6 | | | | |
| ## | 8 | 11 | 75 | 18.2 | | | | |
| ## | 9 | 11.1 | 80 | 22.6 | | | | |
| ## | 10 | 11.2 | 75 | 19.9 | | | | |
| ## | # . | wit | ch 21 mc | ore rows | | | | |



The glimpse function is similar to str but a little friendlier:

glimpse(trees.tbl)

Observations: 31
Variables: 3
\$ Girth <dbl> 8.3, 8.6, 8.8, 10.5, 10.7, 10....
\$ Height <dbl> 70, 65, 63, 72, 81, 83, 66, 75...
\$ Volume <dbl> 10.3, 10.3, 10.2, 16.4, 18.8, ...



Tibbles are like data frames

Tibbles act like data frames in some ways. Functions such as *summary()* and str() are still useful. For example,

```
str(trees.tbl)
## Classes 'tbl_df', 'tbl' and 'data.frame': 31 obs. of 3 va
## $ Girth : num 8.3 8.6 8.8 10.5 10.7 10.8 11 11 11.1 11.2
## $ Height: num 70 65 63 72 81 83 66 75 80 75 ...
## $ Volume: num 10.3 10.3 10.2 16.4 18.8 19.7 15.6 18.2 22
```

The plot () function



plot (trees.tbl)



This is a scatterplot matrix.

The effect of plot (trees) would have been the same.



Tibbles are not like data frames

The girth of a tree is like its circumference, so we might expect the volume of the tree to be related to the square of girth times height.

Specifically, we might predict volume from girth and height using the following formula:

$$V = \frac{G^2 H}{4\pi}$$

We can calculate this prediction from the given data and see how much error there is.

To do this, we need functions in the *tidyr* and *dplyr* packages.

library(tidyr)
library(dplyr) # or just use library(tidyverse)



<u>Tibbles are not like data frames</u>

trees.tbl <- trees.tbl %>%

mutate(VolumePredicted = Girth^2*Height/(4*pi))

trees.tbl

| ## | # P | A tibbl | e: 31 x | k 4 | |
|----|-----|-------------|-------------|-------------|-----------------|
| ## | | Girth | Height | Volume | VolumePredicted |
| ## | | <dbl></dbl> | <dbl></dbl> | <dbl></dbl> | <dbl></dbl> |
| ## | 1 | 8.3 | 70 | 10.3 | 384. |
| ## | 2 | 8.6 | 65 | 10.3 | 383. |
| ## | 3 | 8.8 | 63 | 10.2 | 388. |
| ## | 4 | 10.5 | 72 | 16.4 | 632. |
| ## | 5 | 10.7 | 81 | 18.8 | 738. |
| ## | 6 | 10.8 | 83 | 19.7 | 770. |
| ## | 7 | 11 | 66 | 15.6 | 636. |
| ## | 8 | 11 | 75 | 18.2 | 722. |
| ## | 9 | 11.1 | 80 | 22.6 | 784. |
| ## | 10 | 11.2 | 75 | 19.9 | 749. |
| ## | # . | wit | :h 21 mc | ore rows | 5 |



Why are the predicted volumes off by so much?

To find the answer, read the help file to find that the Girth measurements are actually diameter measurements in inches.

The other variables are in terms of feet.

Re-doing the calculation with diameter, instead of girth, we have

$$V = \frac{\pi * D^2 H}{4(12)^2}$$

We can calculate this prediction from the given data and see how much error there is:

```
trees.tbl <- trees.tbl %>%
mutate(VolumePredicted = Girth^2*Height/(4*12^2))
```

<u>Tibbles are not like data frames</u>



trees.tbl

| ## | # Z | A tibbl | le: 31 x | x 4 | |
|----|-----|-------------|-------------|-------------|-----------------|
| ## | | Girth | Height | Volume | VolumePredicted |
| ## | | <dbl></dbl> | <dbl></dbl> | <dbl></dbl> | <dbl></dbl> |
| ## | 1 | 8.3 | 70 | 10.3 | 8.37 |
| ## | 2 | 8.6 | 65 | 10.3 | 8.35 |
| ## | 3 | 8.8 | 63 | 10.2 | 8.47 |
| ## | 4 | 10.5 | 72 | 16.4 | 13.8 |
| ## | 5 | 10.7 | 81 | 18.8 | 16.1 |
| ## | 6 | 10.8 | 83 | 19.7 | 16.8 |
| ## | 7 | 11 | 66 | 15.6 | 13.9 |
| ## | 8 | 11 | 75 | 18.2 | 15.8 |
| ## | 9 | 11.1 | 80 | 22.6 | 17.1 |
| ## | 10 | 11.2 | 75 | 19.9 | 16.3 |
| ## | # . | wit | ch 21 ma | ore rows | 5 |



The code below causes the errors or *residuals* to be plotted against the predicted volumes as below.





Note that we are still systematically under-predicting the volume and the prediction error is increasing with diameter.

References



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